Remote Sensing of Ocean Sound Speed Profiles by a Perceptron Neural Network

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Abstract—A method is developed to estimate ocean sound speed profiles through synthesis of remotely measured environmental data and historical statistics of sound speed obtained at the remotely sensed location. Sound speed profiles are represented by an expansion of empirical orthogonal functions (EOF) of the historical sound speed variation, while the remotely sensed environmental data provide real-time information to determine the expansion coefficients. Environmental inputs are limited to sea surface temperature available from satellite infrared sensors, acoustic time-of-flight and ocean bottom temperature measurable from bottom mounted acoustic and thermal transducers. A multilayer perceptron neural network is implemented to learn the functional transformation from the measured environmental input to the desired EOF coefficient output on a set of representative sound speed profiles. Sea surface temperature, time-of-year, and time-of-flight from the acoustic multipath that maximally samples the vertical sound speed are found to be the dominant inputs. The trained network is computationally efficient and produces estimates for untrained environmental inputs with a mean error of 1.1–4.4 m/s.

I. INTRODUCTION

In ocean acoustics applications, uncertainty in quantification of environmental parameters often presents a fundamental limitation to the utility of the acoustic measurement/prediction problem at hand. In the case of acoustic vehicle tracking, accurate spatio-temporal quantification of the sound speed profile (SSP) is crucial to the quality of the tracking solution. It is also accepted that accuracy of matched field processing structures are intimately tied to the environmental mismatch of the replica field. Often, the only resource available to sample the acoustically important environmental parameters relies on deployment of mechanical sensors such as expendable bathythermographs (XBT’s) or conductivity-temperature-depth (CTD) probes. To obtain at-sea environmental data over a large spatial extent within a small time window with such mechanical devices is inherently costly and potentially hazardous in heavy weather. This paper demonstrates the feasibility of realizing a remote-sensing SSP monitoring system for locations where multiple bottom mounted acoustic/temperature sensors are installed, such as at existing deep water and planned shallow water Navy underwater tracking ranges.

The approach is based on integration of remotely sensed environmental data with empirical statistics of the SSP to provide real-time spatio-temporal characterization of the SSP. This is achieved by presenting the remotely measured tomographic and temperature information to a multilayer perceptron trained on historical SSP data. Empirical statistics of the SSP’s relevant to a given locale can be obtained from archival information contained in oceanographic databases. Real-time sampling of the SSP is achieved through time-of-flight observations using ocean bottom acoustic sensors employed in an acoustic tomography structure. Temperature data is acquired by measurement of ocean bottom temperature from a thermal sensor at the hydrophones, and estimated sea surface temperature can be derived from multispectral satellite AVHRR (infrared) data. Representation of the SSP is enhanced by an efficient parameterization using empirical orthogonal functions (EOF’s), which constitute the optimal basis functions for expansion of the SSP in the least-squares sense. The task of the neural network is efficient and accurate prediction of the EOF expansion coefficients based on real-time measurements characterizing the state of the environment.

Application of neural network architectures to the parameter estimation of complex multivariate remote sensing problems has emerged recently as an attractive alternative to classical parameter estimation techniques. In a general sense, a neural network can be defined as an algorithm which accepts an n-dimensional input vector, and provides a functional mapping to an m-dimensional output. The neural network is distinct from conventional programming techniques which produce functional transformations in that the relationship between the input and output vectors is not known a priori. The neural network constructs decision boundaries in the parameter space as a result of the network seeking an error minimum during the training process. Application of the trained network then results in an efficient utilization of computational resources in producing parameter estimates.

II. MODEL OF THE SSP

The model used to represent the SSP data is an expansion of orthogonal functions $F_i(z)$ about the background SSP $c(z) = c_0(z) + \sum_{i=1}^{M} \alpha_i F_i(z)$ (1)

where $c_0(z)$ is the mean SSP, and $M$ the number of modal functions. The orthogonal functions of interest are obviously sound speed perturbations with respect to the mean profile. A powerful method of obtaining such functions from a given dataset is the method of empirical orthogonal functions (EOF)
introduced by Kosambi [4]. (See also Davis [5] and Kundu [6].) The efficacy of EOF’s in providing robust estimators for functions of sound speed in the ocean has been exploited by several authors [7]-[9], with typical results that only two to four EOF’s are required for accurate modeling/inversion of the sound speed data.

Empirical orthogonal functions are the eigenvectors of the real and symmetric matrix of correlation coefficients. They are termed empirical since they are constructed entirely from the statistics of the data, and orthogonal because the eigenvalues form a diagonal matrix, ensuring statistical independence between the eigenvectors. In the present case the correlation is one of sound speed between profiles. However, since the expansion functions we seek are ones of sound speed perturbation, we can remove the mean from each of the correlation coefficients and deal with EOF’s of the covariance matrix. Therefore the EOF’s satisfy

$$\tilde{R}F = \Lambda F$$

with eigenvalues $\lambda_{ij} = \lambda_i \delta_{ij}$ where $\delta$ is a Kronecker delta function and where the entries of the covariance matrix are explicitly given by

$$R_{ij} = \frac{1}{N_{SSP}} \sum_{n=1}^{N_{SSP}} \frac{1}{2} \left[ c_n(z_i) - c_n(z_j) \right] \left[ c_n(z_j) - c_n(z_j) \right]$$

with $N_{SSP}$ the number of SSP profiles in the dataset, $i$ and $j$ the depth indexes. The covariance, eigenvalue and eigenvector matrices are therefore computable directly from the data, leaving the specification of expansion coefficients [$a_i$ in (1)] to complete the model.

A. SSP Database

The World Ocean Atlas (WOA) is a comprehensive database of worldwide observed oceanic and interpolated environmental parameters covering the time span from 1900 to 1994. The database was developed at the Ocean Climate Laboratory of the National Oceanographic Data Center (NODC), which is an organization of the National Oceanic and Atmospheric Administration (NOAA) and the National Environmental Satellite, Data and Information Service (NESDIS). The entire database consists of 2.278 Gbytes of observed data, and 962 Mbyte’s of “objectively analyzed” data interpolated from the observed data to standard depths for one and five degree latitude-longitude squares, identified according to area, some 10-30 nmi off the New River/Kamp Legeune area.

The geographic area of interest is located in the Onslow Bay area, some 10-30 nmi off the New River/Camp Legeune area of North Carolina. Specifically, it can be defined by the area contained within 33°-34° N and 76°-77° W. This is a complex coastal ocean environment where the water is composite of the Gulfstream, continental inflow from the nearby rivers and non-Gulfstream coastal water masses.

The WOA observed profile data are analyzed into 10° latitude by 10° longitude “squares,” identified according to the World Meteorological Organization (WMO) numbering scheme. The corresponding WMO square for the Onslow Bay area is numbered 7307. This 10° square portion of the database was searched for CTD/XBT data which fell within the one degree area identified above. In addition to this latitude-longitude restriction, profiles were processed only if they contained valid observed data to a minimum depth of 300 m. Out of the 56,225 XBT profiles that were searched, a total of 520 met the latitude-longitude and depth criteria.

Accordingly, the XBT data were converted into SSP data using the mean salinity of 36.2 ppt via the DelGrosso-Mader equation [11].

The observed CTD data were not sampled at depths which were consistent from profile to profile. Therefore, a standard set of depths was selected for the representation of the SSP’s as follows: from zero depth to 100 m a point was specified every 10 m, from 120 to 300 m a point was specified every 20 m. Each profile was interpolated to these 21 standard depths from the observed data by a natural cubic spline. A sample of 65 resulting SSP profiles which were chosen to uniformly span the January to December time frame is shown in Fig. 1. Note that the SSP structure is predominantly downward refracting.

B. Time-of-Flight

The fundamental physical measurement that forms the basis for prediction of the EOF expansion coefficients in our model
Fig. 2. First seven EOF’s of the 520 SSP’s obtained from the WOA for the Onslow Bay location. The first five EOF’s are labeled, and account for 99% of the total variance in the data. The EOF’s are scaled by the square root of their respective eigenvalues.

is the acoustic time-of-flight (TOF) between two bottom mounted transducers. The TOF is the integral of the sound speed “slowness” over the ray path connecting the source and receiver

$$\tau = \int \frac{ds}{c(s)} \quad (4)$$

and accurate quantification of the TOF in response to variation in $c(s)$ is essential in allowing for sensitive detection of features in the EOF coefficient space. Since the EOF’s we seek are sound speed variations with respect to the mean SSP, the TOF quantity of interest is the time-of-flight difference between the mean SSP and the perturbed SSP, given a fixed source-receiver geometry. A multipath expansion of the wave equation describing the underwater acoustic waveguide was implemented to calculate acoustic time-of-flights. Specifically, the computer software used was the Generic Sonar Model [12].

C. Computation of EOF’s

A program to compute the empirical orthogonal functions from the 520 SSP dataset was written. The symmetric covariance matrix and mean value of the SSP’s were computed for each depth. The Numerical Recipes® [13] function tred2() performed a Householder transformation on the covariance matrix to reduce it to tridiagonal form, and function tqli() then solves for the eigenvalues and eigenvectors. Fig. 2 plots the first seven EOF’s for the SSP dataset. Each EOF is scaled by the square-root of its eigenvalue, thus weighting the amplitude of each EOF corresponding to its strength in contributing to the sound speed perturbation, and providing a dimension of (m/s).

The first five modes (in terms of strength) are labeled in the Fig. 2, and account for the bulk of the data variance. The first and second modes can be observed to dominate the surface layer variations. Exploiting the fact that the eigenvalues represent the portion of variance contributed by each of the modes, Table I quantifies the relative importance of the first five modes in determining the sound speed variations.

<table>
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<th>#EOFs</th>
<th>%Variance</th>
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<tr>
<td>1</td>
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<tr>
<td>2</td>
<td>92.20</td>
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<tr>
<td>3</td>
<td>96.85</td>
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<tr>
<td>4</td>
<td>98.47</td>
</tr>
<tr>
<td>5</td>
<td>99.03</td>
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</table>

Based on this information, it is determined sufficient to truncate the expansion of (1) at $M = 5$. It is theoretically trivial to add additional terms aimed at increasing the accuracy of the representation, however, practically, it is advantageous to use as few terms as possible to minimize the number of expansion coefficients which have to be specified. It is also possible that inclusion of higher-order modes would contribute only measurement errors and not serve to efficiently represent the true variations.

D. Least Squares Estimate of the EOF Expansion Coefficients

In order to provide a set of EOF expansion coefficients which the neural network requires for training, the EOF coefficients are computed for the representative SSP data by a five parameter model fit by least-squares residuals. The problem is this: given an observed SSP, and the first five EOF’s computed from the observed dataset, what are the five EOF expansion coefficients that “best fit” the observed data?

Since we have chosen to include only five EOF’s in the SSP estimates, the sound speed variation $\Delta c(z)$ is computed from the EOF expansion coefficients $c_i$ by

$$\Delta c(z) = \sum_{i=1}^{5} \alpha_i F_i(z) \quad (5)$$

from which it is clear that the number of depths in the profile ($N = 21$) exceed the number of parameters to be estimated ($M = 5$), resulting in an overdetermined set of linear equations. That is, there will not exist an exact solution vector $\alpha$. Therefore, the objective is to identify a solution for the vector $\alpha$ that comes closest to satisfying the (5) simultaneously in the least-squares sense. The five parameter model is defined as

$$\Delta c(z) = \sum_{i=1}^{5} \hat{\alpha}_i F_i(z) \quad (6)$$

where $\Delta c(z)$ is the estimated SSP variation computed with the estimated EOF expansion coefficients $\hat{\alpha}_i$. The best estimates of the five parameters are implemented by minimization of the residuals: $[\Delta c(z) - \Delta \hat{c}(z)]^2$ where $\Delta c(z)$ is the sound speed
This means that on average, the minimum error RMSSSP of the five EOF's is 0.82 m/s. The estimated Summer profile has an RMSSSP of 0.96 m/s, while the Winter profile RMSSSP is 0.69 m/s. The RMSSSP was computed for each of the representative profiles shown in Fig. 1, and resulted in statistics of

\[
\text{RMS}_{SSP} = \sqrt{\frac{1}{N} \sum_{i=1}^{N} [c(z_i) - \hat{c}(z_i)]^2}.
\]

The estimated Summer profile has an RMSSSP of 0.96 m/s, while the Winter profile RMSSSP is 0.69 m/s. The RMSSSP was computed for each of the representative profiles shown in Fig. 1, and resulted in statistics of

- Number of Profiles \( P = 65 \)
- Mean Error \( \langle \text{RMS}_{SSP} \rangle = 0.82 \text{ m/s} \)
- Standard Deviation \( \sigma_{\text{RMS}_{SSP}} = 0.34 \text{ m/s} \)
- Minimum Error \( \text{RMS}_{\text{min}} = 0.21 \text{ m/s} \)
- Maximum Error \( \text{RMS}_{\text{max}} = 2.10 \text{ m/s} \)

This means that on average, the minimum error RMSSSP that can be expected from estimates of the SSP's by a linear least-squares fit of the five EOF's is 0.82 m/s.

III. MULTILAYER PERCEPTRON TO PREDICT EOF EXPANSION COEFFICIENTS

Neural networks are implementations of computational algorithms that provide a functional mapping from an \( n \)-dimensional input to an \( m \)-dimensional output space. They are loosely based on the information processing structure of biologic neurons, and derive their ability to learn complex mapping from the interconnection structure and the inherent nonlinearity of the individual neurons. All neural networks operate in two distinct phases. The first is the training phase, where the network adjusts its internal parameters in response to training data. Second is the predictive phase, where the trained network responds to input data and produces a functional mapping. Neural networks can be generally classified as either supervised or unsupervised architectures. Supervised paradigms require a "teacher" that produces an error output in response to training data, minimizing the error directs the learning process and adjustment of the network internal parameters. An unsupervised network does not require an external error source, but relies on a rule base to adjust the internal parameters in response to the network output during training. Neural networks can be further classified as either feedforward or feedback types. This refers to the flow of information during the predictive phase of operation. Two important features of neural networks are that they have the ability to generalize, or produce accurate estimates of their functional representation in response to inputs that were not present in the training data, and that an explicit model, or conditional probabilities between the functional input and output are not required.

The perceptron is a supervised feedforward network. It requires an error source for the network output in response to training data during the learning phase. It is also a feedforward architecture, and as such is a computationally efficient estimator in the predictive phase. We implement a multilayer perceptron to predict the EOF expansion coefficients based on the 65 observed SSP's extracted from the WOA for the Onslow Bay site. The network calculates the synaptic weight modifications via backpropagation.

A. Multilayer Perceptron Architecture

The network consists of: seven input units, 20 units in the first hidden layer, 20 units in the second hidden layer, and five output units as depicted in Fig. 4. The input units are represented as circles and simply serve as branch out points for each of the input values to the each of the units in the first hidden layer. Between each of the units in successive layers is a weight represented by a solid line. Units in the hidden and output layers are depicted as a squares, and perform computations as follows: Let \( X_i = \sum w_{ij}x_j \) represent the weighted sum of the multiinputs \( x_j \) with \( w_{ij} \) being the weighting factor across the interconnection between the \( i \)th and \( j \)th cells. This summed input is processed by the activation function \( F_S \) to produce the neurons output signal \( O_i \) as depicted by Fig. 5. The activation function provides each neuron with a nonlinear transfer function, allowing for the processing of large input values without overload, while
Fig. 4. Neural network topology of the multilayer perceptron to predict values of the EOF expansion coefficients.

Fig. 5. A computational unit of the multilayer perceptron.

B. Multilayer Perceptron Training

The objective of the training process is to allow the network to learn the functional mapping of the input data to the desired output vector. This is achieved by repeatedly presenting to the network a known set of input/output pairs (training sets), and adjusting the weights to minimize some measure of error between the desired output and the computed network output. In the case of the multilayer perceptron, a conventional error minimization approach is backpropagation [14, [15]. The fundamental quantity used in determination of the weight adjustment is the error or distance $D$ of the network output $O_i$ of the $i$th unit from the target value $T_i$. It is used to calculate the effective gradient of the weight modification term in the backpropagation algorithm. The error metric employed here is the Euclidean distance specified by $DE = T_i - O_i$, and adjustment of weights at the $i$th training step is prescribed by the Widrow–Hoff delta rule [14]

$$w_{ij}(n) = w_{ij}(n - 1) + \eta \delta_j O_i$$

(8)

where $\eta$ is the learning rate and $\delta_j$ is the effective gradient.

C. Training Sets

Aside from selection of the network architecture, identification of a suitable training set which encompasses the relevant feature vectors of the transformation to be learned is the most critical step of network design. Addressing this problem involves conducting a sensitivity study to identify which parameters available in the training design can be effectively utilized by the network to characterize the desired transformation. Another concern is the sensitivity of the network to the numerical magnitudes of the training set. For example, if the desired output values of the network are all close to zero, then the learning rule (8) will not be effective since the magnitude of the weight change is proportional to the value of the network output. Also, if the value of the network inputs are close to zero, the weights connecting these inputs to subsequent processing units will have little consequence. In fact, clamping the input value of a parameter to zero is a method employed in the sensitivity study to selectively remove network inputs without reconfiguring the network architecture.

The input training data available in the present case is limited by the measurement resources available in the operating area. This study assumes that the measurements available are: 1) Time quantified as a day-of-year in the interval [1–365], 2) ocean bottom temperature, 3) sea surface temperature, and 4) acoustic time-of-flight between two bottom mounted transducers. Further, to conform to the average depth of the area under consideration, the bottom depth is taken as 300 m and the sensor separation as 5000 m. Computation of acoustic eigenrays from the mean and representative SSP’s reveal that four acoustic paths dominate the multipath structure for much of the year: 1) a refracted direct path designated as $[1 V 0]$, the 1 V indicating an upper vertexing ray, and the zero referring to no bottom vertexes or reflections, 2) a surface reflected ray $[1 0]$, 3) a surface-bottom-surface reflected path $[2 1]$, and 4) a bottom reflected path with two upper vertexes $[2 V 1]$. The objective of the network implementation is to predict the EOF expansion coefficients of (3) which are the coefficients of sound speed variation with respect to the mean SSP. Accordingly, the mean values of temperature, and time-of-flight for each acoustic path contribute only a bias term to the respective input feature vectors, therefore, these mean values are removed from the input temperature and time-of-flight data. The complete set of input features available is therefore a vector of length seven consisting of the time, two temperature variations, and four acoustic TOF variations.

Out of the 520 SSP’s that met the latitude/longitude and depth criteria, a subset of 65 profiles were chosen to uniformly...
span the January to December time frame (Fig. 1) and are intended to form a suitable basis for a training set. The seven input parameters were extracted from the 65 SSP's as follows: The time parameter should conform to a cyclic function of period one year. Several periodic functions were evaluated for this purpose: a ramp, sawtooth, and sine wave. The sine wave function produced the lowest RMS error of network training output for equivalent network configurations. Therefore, the time is parameterized as time-of-year = sin(2π day-of-year/365). The temperature variations were extracted from the observed XBT data at the surface and bottom points. The magnitude of the temperature variations fell within ±6°C. The multipath TOF's were computed by the Generic Sonar Model for each of the 65 SSP's. The mean SSP (of the 520 computed SSP's) was used to compute the mean TOF for each multipath, which was removed from the 65 training set TOF's to produce the multipath TOF variations (ΔTOF). The magnitude of the TOF variations were less than 0.04. Based on the small magnitude of these variations, the ΔTOF's were scaled by a factor of 100 to span the interval ±4.

Fig. 6 plots the time-of-year parameter and temperature variations versus the day-of-year for the 65 SSP's of the network input training set. The surface temperature variation resembles a periodic function of time, with the maximum and minimum variations following the summer and winter seasons. Fig. 7 presents the scaled ΔTOF's for each of the dominant multipaths. It is difficult to discern any clear pattern of temporal variation from this data, although the surface reflected path (1 0) does indicate a somewhat cyclic pattern. Note that the higher order multipaths (2 1) and (2 V 1) are not present in all of the data. Figs. 6 and 7 constitute the complete set of network input training data available to the network.

The network output training set consists of the five EOF coefficients associated with each of the 65 observed SSP's. The EOF coefficients were solved as a five parameter linear least-squares fit of the EOF basis functions to the 65 observed SSP's. Owing to the large variance of the low order EOF's, the resulting magnitude of coefficients was in the range of [−0.1−0.15]. To avoid training the network to these small target values, the EOF's were scaled by 0.01, resulting in the coefficients being scaled by a factor of 100. Fig. 8 plots the resulting EOF coefficients and constitutes the network output training set (target values). Except for the sea-surface temperature, and marginally for the reflected acoustic time-of-flights, the environmentally derived training data does not exhibit the temporal regularity that one might expect. This may be a reflection of the dynamic nature of the interacting water masses in Onslow Bay.

D. Sensitivity Study

The objective of the sensitivity study is to identify the information content of the input parameters in relation to the networks ability to learn the output transformation as efficiently and accurately as possible. It is conducted by training the network with various combinations of the input parameters and identifying the combination which produces the fastest convergence and minimum output error. Another goal is to identify whether or not inclusion of additional training information can improve the network performance based on inputs that are not included in the training data. That is, what effect does the information content of the training set have on the ability of the network to generalize based on what it has learned from training? The generalization capability is of central importance since the utility of the network ultimately lies in producing accurate estimates based on unknown (untrained) input data. This is accomplished by training the network with subsets of the training set, and observing the trained networks output error for untrained inputs. Each of these endeavors might identify changes to the network architecture. The former will indicate if any of the input units can be removed from the network, while the latter may mandate increasing the hidden layer structure and
number of intermediate weights to accommodate learning a larger training set.

1) Input Parameter Sensitivity: To perform the input parameter sensitivity study, the contribution of an input parameter is controlled by either obtaining its value from the input training set or clamping its input value to zero. The weights and thresholds are initialized to pseudorandom uniform values over [-1, 1], and this same set of random initializations is used for each network configuration. A learning rate coefficient of 0.002 is used. Since the purpose is to assess the utility of each of the input parameters to the learning process, it is not necessary to fully train the network for optimum prediction performance which typically requires thousands of iterations. It is only required that the error minimization seek out the local parameter space of the global error minimum, which was observed to occur within 200 iterations for all network configurations. The network is therefore sequentially presented the 65 training sets for 250 iterations. The RMS error between the desired output and network computation (RMS$_a$) is recorded for each input parameter configuration at the conclusion of training. This error quantifies the average deviation of the $M = 5$ predicted EOF expansion coefficients from those of the target set, averaged over the $P = 65$ training sets as follows

$$\text{RMS}_a = \sqrt{\frac{1}{PM} \sum_{i=1}^{P} \sum_{j=1}^{M} (\alpha_{ij} - \hat{\alpha}_{ij})^2}.$$  

(9)

Table II summarizes the input parameter combinations and resultant RMS errors in prediction of the five EOF expansion coefficients. A check mark indicates that the input parameter was included in the network configuration.

The first line of Table II represents the result of presenting the full input training set to the network. Lines 2–5 sequentially remove the tomographic information presented as input for the four acoustic multipaths. Lines 6 and 7 indicate results for removal of the reflected and refracted eigenrays respectively, while lines 8–10 isolate effects of withholding the time-of-year, bottom and surface temperature variations respectively.

Inspection of the RMS errors as a function of the tomographic input (rows 2–7) reveals two important conclusions. First, removal of the TOF variation for the surface-bottom-surface [2 11 reflected eigenray has the greatest detrimental effect on network accuracy. This is expected since this acoustic path provides the most complete sampling of the vertical SSP structure.

Second, removal of the refracted bottom-reflected path [2 V 1] results in the smallest RMS prediction error of any of the input parameter configurations. This means that the tomographic information contained in this eigenray was serving to confuse the network in learning the desired functional mapping. It indicates that the data from the [2 V 1] eigenray is adding more noise than information in deciding the optimal network output. This is consistent with the fact that this eigenray propagates nearly horizontally, and samples on average only the lower 33 meters of the water column, thereby missing information contained in the crucial near surface area.

The results presented in lines 8 and 10 provide additional insights to the sensitivity of the network inputs. They indicate that the surface temperature variation is the single most influential input parameter in the learning, while the time-of-year also contributes largely to accurate functional mapping. Based on the results presented in lines 2–10, combinations of input parameters which excluded the less important inputs were tried in order to identify if a reduction in input parameters could be found that would not adversely affect the network error. The results of three such combinations are presented in lines 11–13, where the reduction in inputs only served to increase the network error.

The results of the input parameter sensitivity study reveal that the bottom refracted tomographic input available from the complete training set was increasing the RMS error of the network output. Accordingly, the network was reconfigured
to accept an input vector of six parameters, and all following results were obtained from such a configuration.

2) Training Set Sensitivity: To investigate the information content of the representative SSP's selected for the network training set, the network was trained for 3000 iterations with identical weight and threshold initializations for the full 65 SSP training set, and subsets consisting of 55, 45, and 35 training sets selected at random. Six profiles which were not included in the training set were chosen from the database and were used to generate input data for the trained networks. The six profiles were selected to span the entire year, and are delineated by the day-of-year on which they were measured (45, 108, 154, 205, 267, and 327). The trained networks were then presented the input data from the six "unknown" profiles, and used to estimate the corresponding EOF expansion coefficients. The predicted SSP's were constructed from the EOF's, and the RMS error of the estimated profile with respect to the observed data (RMS$_{SSP}$) calculated. These errors are presented in Table III along with a comparison to the error of the five parameter linear least-squares fit of the EOF's to the observed profiles.

The results of Table III show that averaged over the six untrained profiles, the RMS error in prediction of SSP's decreases as the number of training sets is increased. This indicates that the training set may have not yet reached its full potential in providing information usable to the network concerning the transformation of the statistical and environmental inputs into appropriate EOF coefficients. It is also observed that the error for the full training set condition is roughly two to five times that of a five parameter linear least-squares fit of the EOF's to the SSP's. This also suggests that the network training set is incomplete. However, it can be seen that use of the full training set does not ensure the minimum error prediction for each of the profiles. Specifically, the profile of day 205 was optimally predicted by the network trained with 55 sets instead of the available 65. This illustrates the sensitive interdependence between the generalization capabilities of neural networks and the ability of the network to accurately learn the desired functional transformation based on the information content of the training data.

### Table III

<table>
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<th>Number of Training Sets</th>
<th>35</th>
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<td>Day</td>
<td></td>
<td></td>
<td></td>
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<tr>
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<tr>
<td>Avg RMS$_{SSP}$</td>
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Fig. 9 presents SSP predictions for six sets of input parameters derived from profiles that were not included in the training set. The network was trained for 3000 iterations with the full training set, and the resulting RMS error of the five EOF expansion coefficients over the 65 training profiles was RMS$_{d}$ = 0.077. The error in sound speed (RMS$_{SSP}$) for each of the profiles is tabulated in the 65 training set column of Table III.

### IV. Conclusion

A multilayer perceptron neural network has been implemented to predict ocean sound speed profiles based on the input of remotely sensed acoustic tomographic, and ocean boundary temperature data. Empirical orthogonal functions of sound speed variation were computed from a historic sound speed profile database, and a truncated series of the orthogonal functions form the basis of the predictive model. The remotely sensed environmental data specifies the boundary conditions, and allows the trained network to estimate the series coefficients based on generalization of the learned input/output transformation.

An attractive aspect of the perceptron neural network is the computational efficiency with which SSP predictions can be made once the network is trained and the input data for a prediction are collected. All that is required is to present the input data to the trained network, process one forward sweep through the network, and implement the EOF expansion coefficients to construct an SSP estimate. The computational burden arises from training of the network, during which feedback error from the network output is minimized over...
the training set. Construction of the training set may require significant work to process and identify the sensitive input parameters, and may serve to adjust the configuration of the network itself.

Network predictions on untrained input parameters produced RMS errors over the depth of the profiles in the range of 1.1–4.4 m/s. A linear least-squares fit of the basis functions to these profiles resulted in RMS errors from 0.7 to 1.3 m/s, indicating that the network architecture/training data were not fully optimized to the desired transformation. A sensitivity study of the information available in the network training set revealed that the surface temperature, time-of-year, and acoustic time-of-flight from the eigenray which maximally samples the vertical sound speed structure were of primary importance in learning the desired transformation. It was also shown that the time-of-flight from a downward refracted bottom reflection path was serving to confuse the network and decrease the training efficacy.

The above results demonstrate that realization of real-time SSP estimates in a complex coastal environment based on input of remotely sensed data to a neural network are feasible, however, two limitations related to the physical significance of the proposed method should be mentioned. First is the assumption that the mean values of temperature and acoustic time-of-flight which were removed from the training data are stable. Temperature dependent geophysical ocean parameters are expected to have time-dependent means, and so the significance of assuming that the means were stable over the period of the historic database should be assessed. Second is the use of a range-independent acoustic model for computation of the time-of-flights. Since the site investigated in this work is near both the continent and the Gulfstream, a range dependent model which could incorporate sea-surface temperature and bottom tomography variations between the acoustic sensors should be explored.

REFERENCES