Sigmoidal Activation of Proportional Integral Control Applied to Water Management

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Abstract: A conventional proportional integral (PI) controller is modified with a nonlinear activation function (sigmoid function) applied directly to the controller output in order to improve the stability and target fidelity of the system response to large variational inputs in both state and internal controller gain variables. The controller is implemented in a simulated water management role applied to a major subregional pumping station between Lake Okeechobee, the Loxahatchee National Wildlife Refuge (WCA1), and the West Palm Beach supply canal (C51) in the South Florida geographic region. The simulation consists of an integrated hydrological numerical model implemented in the Regional Simulation Model (RSM), which is currently under development at the South Florida Water Management District. Analysis of the modified controller in the Laplace domain establishes the expected control behavior, and subsequent results of the simulation for the conventional PI and the sigmoid modified controller are presented and compared. The modified controller achieves significant improvement in stability while simultaneously reducing the control signal energy.

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Introduction

Industrial control technology applied to water resource management has achieved wide applicability in many regions. Such water management control applications have some unique control system parametric regimes owing to the spatially distributed systems containing significant masses of water. Coupled with this is a large potential variation in state variable and forcing function values in response to storms or unusual climatic events. Most conventional industrial control algorithms are "tuned" process-control state machines, and can be sensitive to changes in parametric state space conditions, as well as to internal controller gains. It is possible that even small variations of these variables can precipitate controller destablization and loss of performance if the environmental inputs or controller gain parameters exceed the boundaries of the expected control regime. In the case of water management control, failure to adequately control a local, subregional, or regional watershed may result in substantial economic, environmental, or water quality impacts. In an attempt to mitigate such circumstances, the development of robust controller algorithms that achieve good transient and steady-state responses in the face of large variability of state parameters is an area of pragmatic concern. The focus of this paper is the development of a modified PI controller that exhibits improved stability and gain response characteristics while conserving the energy of the control commands.

A variety of control algorithms and systems have been proposed and applied to individual water management structures (Buyalski 1991; Rogers and Goussard 1998), but have been primarily associated with canal control structures. At the other end of the spectrum, a large body of work has investigated the optimization of reservoir resource routing, wherein the control characteristics of the individual management structures are typically not explicitly considered. Additionally, many of these advances may fail to couple conjunctive-use aquifer/stream interaction with the regional water policy decisions (Belaineh et al. 1999). All of these issues are facing the South Florida Water Management District in the development of the Regional Simulation Model (RSM), a comprehensive, new-generation hydrological model intended to serve the numerical modeling needs of the district and the federally mandated Comprehensive Everglades Restoration Plan (WRDA 2000).

The RSM consists of two interactive, primary components—the hydrologic simulation engine (HSE) and the management simulation engine (MSE). The HSE is a finite-volume, coupled surface/groundwater/canal numerical model with full two-dimensional and partial three-dimensional flow capability. HSE includes structure flow equations for a wide variety of control structures, and implements efficient numerical solutions of conjunctive hydrological simulations (Lal 1998). The MSE is comprised of two primary subcomponents: a suite of low-level structure control algorithms that serve as flow regulators for individual

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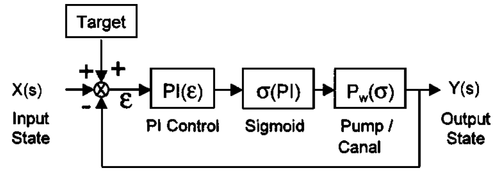

Fig. 1. Sigmoid control system

structures (Park et al. 2003a), and, a set of high-level supervisory control functions that provide dynamic controller modification and coordination intended to facilitate regional control objectives (Park et al. 2003b). The low-level controllers include a generic proportional-integral-derivative (PID) controller, a PI modified sigmoid controller (as described in this paper), a user-defined piecewise linear transfer function controller, a rule-based expert system (fuzzy) controller, and a user-defined finite-state machine controller. The supervisory control methods currently include a linear programming (LP) optimization and a rule-based expert system characteristic field controller (fuzzy control supervisor.) The high-level supervisory control functions are not the topic of this paper, but will be addressed in future work.

In the present analysis a single-input/single-output, sigmoid modified PI controller is applied to a major subregional pumping station (S5-A) that regulates water levels in a feature-rich locale at the confluence of a major Lake Okeechobee drainage canal, a major urban supply canal, and an environmentally sensitive conservation area. The primary forcing function is the regional precipitation, which can exhibit significant temporal/spatial amplitude variations, particularly in the seasonal monsoon climate. The control objective is to maintain water stage levels in the Loxahatchee National Wildlife Refuge conservation area canals near a constant target value. To assay the performance of the controllers in response to significant forcing, the simulation period selected is the historically active precipitation during the summer of 1988. Results are presented that demonstrate that the modified PI control algorithm can tolerate large variations in controller gain parameters and environmental state conditions while preserving control stability.

Sigmoid Controller

The sigmoid controller is a PI controller with a single nonlinear activation function (the sigmoid) directly filtering the control output of the PI controller. State equations for a generic PID control system, as well as the discrete time implementations of the PI and sigmoid controllers of the MSE are given in the Appendix. Fig. 1 shows a schematic of the sigmoid controller, where $X(s)$ denotes the system input function corresponding to water level state changes resulting from external forcing such as rainfall, and $Y(s)$ refers to the output (downstream) stage. The water stage error (deviation from target stage, T) is $\epsilon = T - Y(s)$, which serves as input to a conventional PI controller. The PI controller output is fed as input to the sigmoid function $\sigma(\text{PI})$, which in turn provides output control signals to the control system pump $P_w(\sigma)$.

Implementation of the sigmoid controller is achieved as follows: let $h_{\text{PI}}(t)$ represent the time domain system function of a PI controller. The sigmoid controller processes the PI output with the sigmoid function, and scales the result by a constant scale factor α. The resultant sigmoid control signal is therefore given by

$$h_\sigma(t) = \alpha\sigma[h_{\text{PI}}(t)] \tag{1}$$

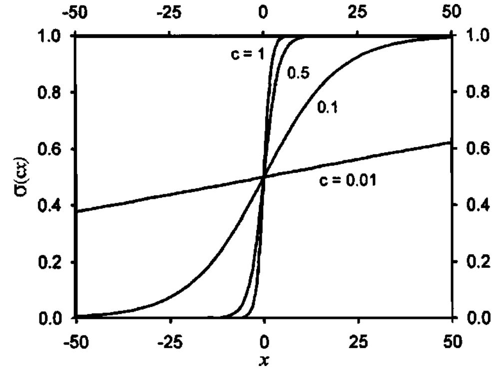

Fig. 2. Sigmoid activation function

The sigmoid function is a bounded function with limits at $-\infty$ and $+\infty$ of 0 and 1 respectively. The sigmoid function can be expressed as

$$\sigma(cx) = \frac{1}{1 + e^{-cx}} \tag{2}$$

with $c > 0$. The derivative is specified by $\sigma'(cx) = c\sigma(1-\sigma) > 0$, from which it follows that σ is a smoothly increasing monotonic function. A plot of $\sigma(x)$ is shown in Fig. 2 for several values of the positive constant c. The value of c determines the slope of the function at the origin, and can change the functional behavior from that of a slowly rising transition $(c \to 0)$ to one of a unit step function $(c \to \infty)$.

Variants of the hyperbolic tangent, or sigmoidal functions, are commonly employed as the neuronal activation function in neural networks. The sigmoid controller is therefore analogous to a PI controller with a single output neuron modulating the control function.

Sigmoid Control Response

In order to understand the control function modifications introduced by inclusion of the sigmoid function as a control stage filter to a generic PI controller, one may examine the Laplace transform of the sigmoid function. The Laplace transform of Eq. (2) can be expressed as

$$\sum(cs) \equiv L[\sigma(ct)] = \frac{\Psi\left(\frac{s}{2c}\right) - \Psi\left(\frac{c+s}{2c}\right)}{2c} \tag{3}$$

where $\Psi(z) = \Gamma'(z)/\Gamma(z)$ is the digamma function, with $\Gamma(z) = \int_0^{+\infty} t^{z-1} e^{-t} dt$. A plot of the $\Sigma(cs)$ magnitude is shown in Fig. 3.

As expected from the exponential basis of $\sigma(t)$ and $\Sigma(s)$, the sigmoid control function $\Sigma(s)$ defines a complex exponential curve, which by definition of the Laplace transform, is a bounded function. The functional nature in the control state domain is essentially a nonlinear low pass filter. Control state variables that are small in magnitude will have a medium to large gain applied, thereby providing enhanced sensitivity to fine-scale control adjustments. In the large control state regime, the filter suppresses large changes in control output, thereby stabilizing the control response. For example, the unity gain point $\Sigma(s) = 1$ occurs at a control state of $s = 0.73$ with $c = 1$, beyond which the control gain diminishes exponentially. The sigmoid control function therefore provides a control stage that provides enhanced control sensitivity to control regions that are inherently controllable, while suppress-

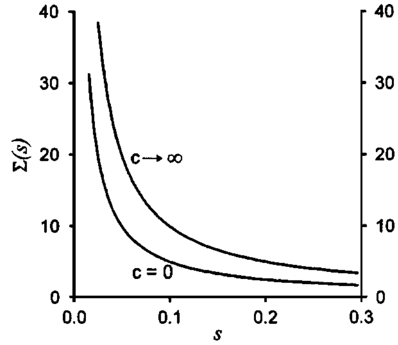

Fig. 3. Laplace transform of sigmoid function

ing control actions that tend toward divergence. The adjustable parameter c enables the user to control the slope of the sigmoid at the zero control point, which translates into adjustment of the sigmoid control function amplitude rise and decay characteristics.

In the event that the control outputs in the interval of $[0,1]$ are not desired, one could add an amplitude bias term to the sigmoid, for example: $\sigma(cx)=1/1+e^{-cx}-1/2$, which would shift the output limits to $-1/2$ and $+1/2$, with zero output at the equilibrium (zero PI control output) point. The corresponding control response function then becomes $\Sigma(cs)=\Psi(s/2c)-\Psi(c+s/2c)/2c-1/2s$ where the effect of the additional term is to shift the response curves toward the origin in the control plane without changing the shape of the curves. The bias terms can therefore be used to provide scaling of the control response.

Simulation Environment and Control Objective

In order to identify performance characteristics of the sigmoid controller under demanding, real-world conditions applied to subregional water management policy, an RSM simulation of the Arthur R. Marshall Loxahatchee National Wildlife Refuge [Water Conservation Area 1 (WCA1)] was applied.

The refuge contains one of three water conservation areas in South Florida, and is maintained to provide water storage and flood control, as well as habitat for native fish and wildlife populations. The refuge encompasses the remaining northern Everglades. Nonrainfall water inputs to the refuge are regulated by a series of pumps, canals, water control structures, and levees built by the U.S. Army Corps of Engineers. The entire refuge comprises about 596 sq km (147,392 acres) and is surrounded by a 92 km canal and levee. The refuge is home to the American alligator and the endangered Everglades snail kite. In any given year, as many as 257 species of birds may use the refuge's diverse wetland habitats. The conservation area therefore is environmentally sensitive, and close control of the incumbent water levels are important to the health and ecological viability of the refuge. A recent LandSat composite satellite image of the South Florida area is shown in Fig. 4. Labeled features include: Lake Okeechobee (LOK), Loxahatchee National Wildlife Refuge (WCAI), urban areas (URB), agricultural areas (EAA), and Everglades (EVG).

The spatial information of the model is encoded into a finite-element grid, with triangular grid cells quantizing the flow areas for surface and groundwater flows. The model consists of 16,292 cells, with a mean cell perimeter of approximately 790 m (2,600 ft). The canals, levees, and other water-control structures are modeled as spatially discrete watermovers with physical at-

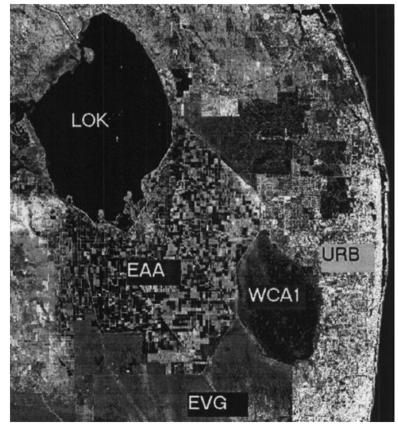

Fig. 4. LandSat of South Florida model region

tributes that closely match the actual structures. A depiction of the model canal perimeter, with significant control structures, is shown in Fig. 5.

The primary forcing function in relation to S5-A pumping activity and therefore, water stages in the refuge, is regional rainfall. There are both seasonal and episodic (storm) components that influence the tributary stages at any point in time. The time period used in the simulation runs from July 1 to September 15, 1988. A representative daily local rainfall timeseries for this period is shown in Fig. 6. A cumulative monthly value of rainfall for the refuge and the area directly north of the refuge reveals values in the range of 25–30 cm (12–14 in.) for the months of July and August. It is therefore consistent to expect large flow volumes as requisite control states for the S5-A pumping stations during this timeframe.

Sigmoid Controller Application

Control of the water stage within the refuge is largely achieved through operation of the S5-A pumping station. This structure is a six-unit pumping plant located at the northernmost point of the refuge. Pump inflow or outflow is into a main canal that splits to feed the eastern and western canals that encompass the refuge. Each pump is rated at 22.6 m^3/s (800 cfs), with a total station

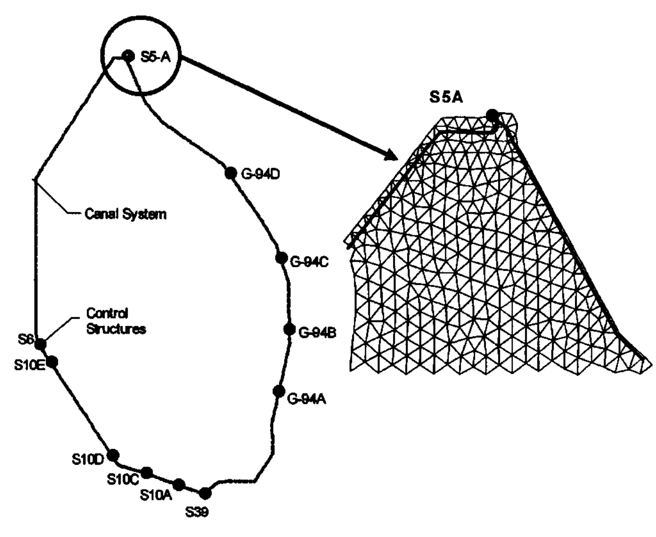

Fig. 5. Regional simulation model schematic

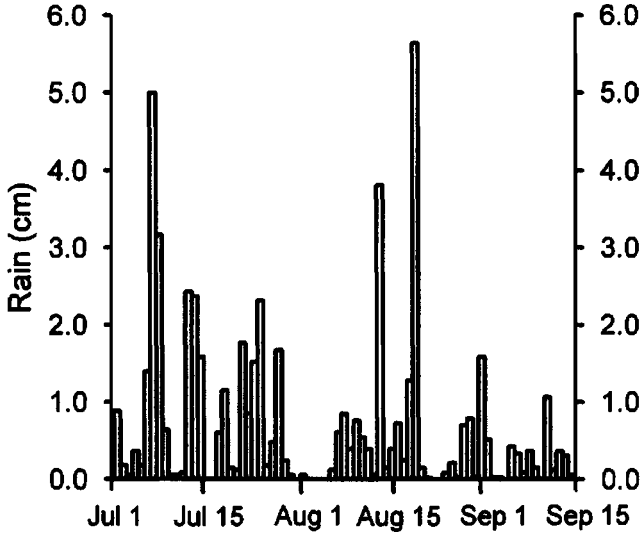

Fig. 6. Regional simulation model rainfall

capacity of 130 m^3/s (4800 cfs). S5-A provides recharge to WCA1 perimeter canals, which in turn drive the WCA1 interior stage levels owing to overland flow from the canal perimeters inside the refuge, as well as the significant porosity of the South Florida soils and surficial aquifers. In the simulation presented here, the aggregate flow rates for the S5-A station are controlled in response to seasonal rainfall covering the period July 1 to September 15, 1988. It is assumed that the control objective is to maintain a stage of 5.18 m (17.0 ft) in the canal sections directly downstream of the S5-A station. (Note that this control objective is arbitrary, and is defined only for the purpose of evaluation of the controllers in a simulation environment. No extrapolations or interpretations of the resulting water levels, or the objective, can be made as a basis for actual operational guidance or directives.) Numerical computations to solve the system hydrological response are performed by the HSE component of the RSM. The controller receives as input the downstream stage value computed by the HSE, and returns as output a value in the interval [0,1], which is mapped to pump station flow required to achieve the control objective. Within the HSE, the pump station flows are specified through linear interpolation of a lookup table as shown in Table 1.

Notice that in Table 1, maximum positive flow into the system is achieved when the sigmoid control output is 0. Likewise, the maximum negative flow removed from the system occurs at the sigmoid limit state of 1, while the equilibrium position of no pump flow corresponds to the sigmoid output of 0.5. As deduced by inspection of the sigmoid function in the Laplace domain, this configuration will result in good control sensitivity near the equilibrium regime, while providing limited control excursions in response to large flow states.

Table 1. Pump Control Table

Control	Flow (m^3/s)	Flow (cfs)
0.0	130.3	4600.0
0.1	104.2	3680.0
0.2	78.2	2760.0
0.3	52.1	1840.0
0.4	26.0	920.0
0.5	0.0	0.0
0.6	−26.0	−920.0
0.7	−52.1	−1840.0
0.8	−78.2	−2760.0
0.9	−104.2	−3680.0
1.0	−130.3	−4600.0

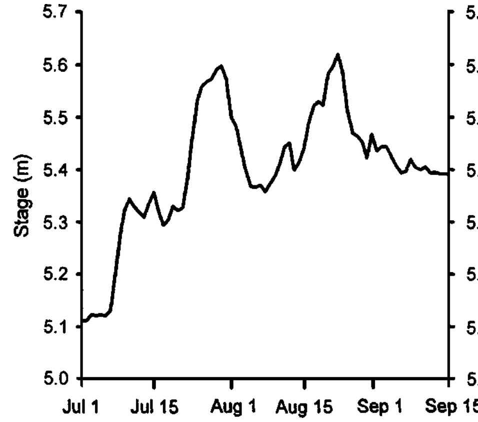

Fig. 7. System response: no control

No Control

In the absence of feedback control, the response of the system to rainfall and the historical pump station flow is used to define a baseline case for comparison of the controller operations. Fig. 7 plots the RSM–computed hydrological stage response at the canal segment downstream of the S5-A station with no applied control. It is observed that levels are consistently in excess of 20 cm above the target value of 5.18 m.

Proportional Integral Control

The stability and fidelity of PI controllers is highly dependent on the component gains for a given state configuration. Further, without knowledge of the compensated system control function, determination of these gains can be problematic. Based on the wide application experience of PI control, several prescriptive algorithms have been developed to address this issue. Here we employ the Zeigler–Nichols method to tune the controller for

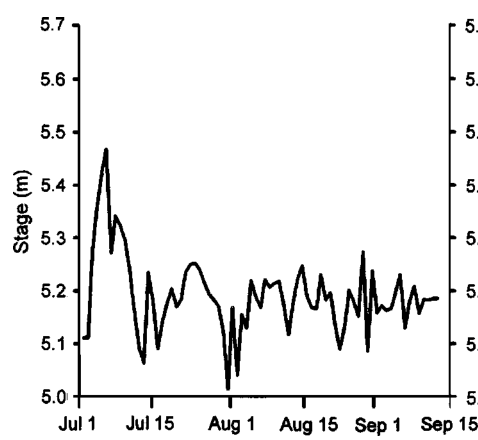

Fig. 8. System response: tuned proportional integral control

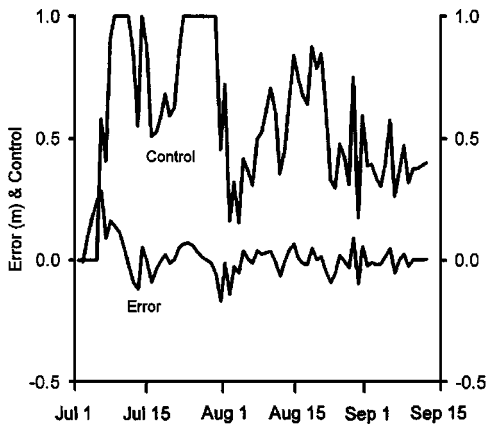

Fig. 9. Control and error: tuned proportional integral control

single-period, 25% amplitude decay. (The Zeigler–Nichols tuning method is employed as follows: Set the integral gain term to zero. Gradually increase the proportional gain from zero until the system just begins to oscillate continuously. The proportional gain at this point is the ultimate gain, P_U. The period of oscillation at this point is the ultimate period, T_U. Set the proportional and integral gain values according to: $\gamma_P = 0.45 P_U$; $\gamma_I = 0.54(P_U/T_U)$.) The resulting proportional and integral gain values were: $\gamma_P = 0.585$, and $\gamma_I = 0.000008$. With these gain values applied to a PI controller for the S5-A station, the simulation was run producing state results shown in Fig. 8. It can be seen that the target value is fairly well maintained, with a slight oscillation near the September 1 timeframe.

In order to quantify the performance of the controller in satisfying the target values, let $E(i) = y(i) - T(i)$ define the discrete arithmetic error of the controlled system output and the target value for the ith timestep. The control signal and resultant error for the tuned PI control are shown in Fig. 9.

Sigmoid Control

A sigmoid controller with a gain value of $\alpha = 1$ and a sigmoid parameter value of $c = 0.1$ were implemented and attached to the same watermover (nonconcurrently) as the PI controller. A tuning procedure based on the Zeigler–Nichols approach was used, resulting in PI gain parameters of $\gamma_P = 50$, $\gamma_I = 0.05$. The RSM model simulation results are presented in Fig. 10. The performance of the sigmoid controller is similar to that of the tuned PI controller (Fig. 8), with smaller excursions from the target value.

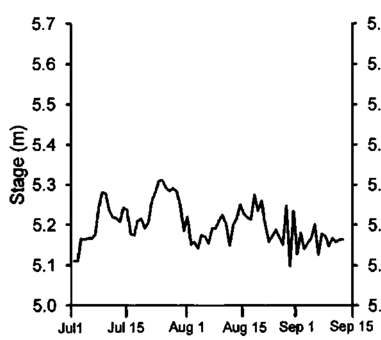

Fig. 10. System response: sigmoid control

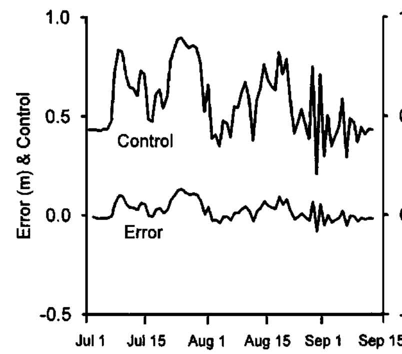

Fig. 11. Control and error: sigmoid control

The error and control signals for the sigmoid controller case are shown in Fig. 11. In comparison to the PI controller (Fig. 9) the controller command excursions as well as the target error signals are smoother and of smaller amplitude than the PI control. The observation that the control signal contains less energy than the PI case could be an important advantage for the sigmoid controller. The expenditure of less control energy implies that less work is done to achieve the control objective, which translates into savings of equipment power and maintenance. From an environmental perspective, it is likely advantageous to minimize the downstream flow and head variations in achieving target stage values.

Control Comparison

As previously noted, conventional PI controllers are sensitive to internal gain variations as well as the parametric regime of the control state variables. To establish the relational performance of the PI and sigmoid controllers, a series of simulations were conducted across a spectrum of gain values. With a fixed value of integral gain ($\gamma_I = 0.000008$ from the tuned PI case) the proportional gain was varied for both controllers over the range of 0.1 to 100. As a comparison metric the RMS error over the timeseries is specified as

$$E_{\mathrm{RMS}}(i) = \sqrt{\sum_{i=1}^{n} \frac{E(i)}{n}} \tag{4}$$

The resulting RMS error of the S5-A downstream stage value was computed for each simulation run, and is shown in Fig. 12.

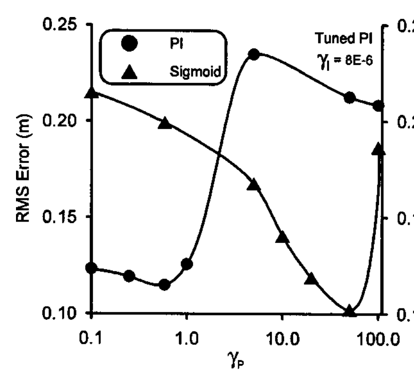

Fig. 12. RMS error of S5-A downstream stage with tuned proportional integral and sigmoid control

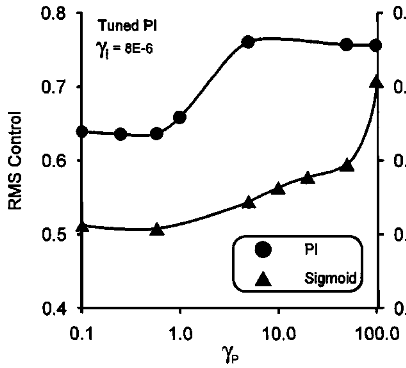

Fig. 13. RMS control of S5-A downstream stage with tuned proportional integral and sigmoid control

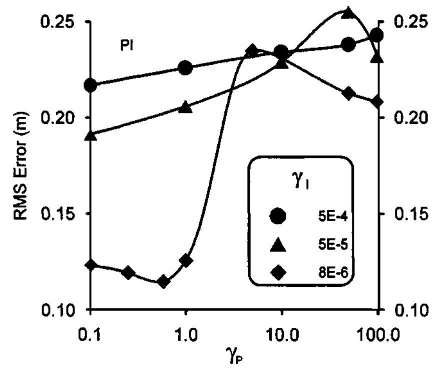

Fig. 14. RMS error of S5-A downstream stage with proportional integral control versus proportional gain with integral gain variation

Both controllers are observed to achieve error minima with respect to γ_P over a fairly narrow range; however, the sigmoid control results in a smaller RMS error than the tuned PI case.

In addition to a smaller RMS error, it was also observed that the control signal itself appeared to be smaller for the sigmoid controller than for the PI controller (compare Figs. 11 and 9). To quantify this observation the RMS control signal was computed for the cases of fixed integral gain ($\gamma_I = 0.000008$) with the proportional gain variations from 0.1 to 100 for both controllers. These control metrics are plotted in Fig. 13.

Fig. 13 suggests that the amount of control exerted by the sigmoid controller is substantially less than that of the PI controller over a wide range of proportional gain.

In addition to the proportional gain, the other important internal gain parameter for PI control is the integral gain. To assay the sensitivity of the controllers to variations in integral gain, the controllers were evaluated over the range of proportional gain for several values of integral gain. The RMS error of the PI control for several values of integral gain is shown in Fig. 14.

Fig. 14 indicates that the conventional PI control is sensitive to changes in integral gain, with substantial increases in system error at tuned values of proportional gain when the integral gain is increased by factors of 6.3 and 63 (from 8E-6 to 5E-5 and 5E-4 respectively). The integral gain response of the sigmoid controller is presented in Fig. 15. Here it is observed that in the region of proportional gain for which the sigmoid controller error is minimal ($20 < \gamma_P < 80$), the RMS error response is largely insensitive to integral gain over four orders of magnitude ($0.0005 < \gamma_I < 5$).

Conclusion

The objectives and constraints placed upon many managed water resource structures make it imperative that effective and reliable control functions can be implemented and maintained. Reliable and stable control requires that the controllers are able to respond gracefully to a wide range of control state inputs, including those that were not implicitly considered in the controller design and tuning. Maintenance of these control functions requires that the respondent control parameters will be functions of time and other state variables, possibly covering a wide range of values. The generally unsatisfactory behavior of conventional PI controllers to such state and control parameter variations provides motivation to design and implement control algorithms that inherit the beneficial qualities of canonical feedback state control systems, while suppressing their undesirable characteristics. The focus of this paper is the implementation of a modified PI controller to address such concerns.

A conventional PI controller was modified with a single nonlinear activation function to improve the stability and gain response characteristics of the controller. The newly devised sigmoid controller was applied to a hydrological simulation involving a major water management pumping station that couples Lake Okeechobee with the environmentally sensitive Loxahatchee National Wildlife Refuge and a major urban water supply and drainage canal. Simulations were performed during the historically difficult period covering the summer of 1988, when above-average rainfall placed significant loads on the management and operation of the pumping station.

Results of the hydrological simulations reveal that the sigmoid controller can achieve lower state variable error response than that of a tuned PI controller. An advantage of the sigmoid controller is the persistent maintenance of acceptable error values over a wide range of internal gain values. Both the sigmoid and PI controllers exhibited good response for proportional gain (γ_P) variations over the range of roughly one order of magnitude. However, the PI control was found to be sensitive to variations in integral gain (γ_I), while the sigmoid control was less sensitive to a wide range of integral gain (5E-3 to 5). The ability to perform useful state control over a wide range of internal gain parameters provides a degree of freedom and error tolerance to the controller designer. It also reduces the burden on the real-time supervisory controller responsible for maintenance of control gains in response to state variable changes. Another attractive feature of the

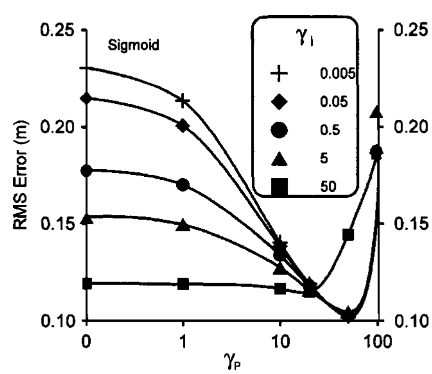

Fig. 15. RMS error of S5-A downstream stage with sigmoid control versus proportional gain with integral gain variation

sigmoid control is expenditure of less control energy as compared to the PI control, resulting in less wear to machinery and likely reduced environmental stress.

Based on the results of the expected and simulated behavior of the sigmoid controller, the sigmoid control algorithm can be considered a viable modification to conventional PI control algorithms applied to water management control operations where decreased sensitivity to internal gain parameters with stable response to large variations in state input is advantageous.

Appendix. Control Implementation in Management Simulation Engine

In the time domain, the PID control can be represented *via* the expression

$$h_{\mathrm{PID}}(t) = \gamma_P \epsilon(t) + \gamma_I \int_0^t \epsilon(t)dt + \gamma_D \frac{d\epsilon}{dt} \tag{5}$$

where γ_P, γ_D, and γ_I=gain factors for the proportional, derivative, and integral terms; and ϵ=system error. Conversion of this expression into a time difference equation results in

$$h_{\mathrm{PID}}(i) = \gamma_P \epsilon_i + \gamma_I \sum_{i=1}^{n} \epsilon_i \Delta t + \gamma_D \frac{\Delta \epsilon_i}{\Delta t} \tag{6}$$

Assuming that a simple arithmetic difference is employed as the system state error metric so that $\epsilon(i)=\phi(i)-T(i)$, where the current system state variable to be controlled is $\phi(i)$ and the desired system target state $T(i)$, the PID control computation for a single time-step is implemented in the MSE as

$$h_{\mathrm{PID}}(i) = \gamma_P(\phi_i - T_i) + \gamma_I \sum_{i=1}^{n} (\phi_i - T_i)(t_i - t_{i-1}) + \gamma_D \frac{(\phi_i - T_i) - (\phi_{i-1} - T_{i-1})}{(t_i - t_{i-1})} \tag{7}$$

As described earlier, the sigmoid controller is a PI controller with a modified output control stage. The PI portion of the controller for discrete time steps is computed as shown in Eq. (7) with $\gamma_D=0$. This control output is filtered by the sigmoid, and scaled by a constant multiplicative factor to produce the sigmoid control output

$$h_\sigma(i) = \alpha\sigma[h_{\mathrm{PI}}(i)] \tag{8}$$

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